Bone Mechanics

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References and Acknowledgments

(In addition to the ones mentioned in Bone Anatomy and Physiology)

BOOKS

- Martin R.B. et al. *Skeletal Tissue Mechanics*. 2015
- Bartel B.L. et al. *Orthopeadic Biomechanics*. 2006

JOURNAL ARTICLES

- Several – see references on slides

PHD THESIS

- Helgason B. *Subject Specific Finite Element Analysis of Bone With Particular Application in Direct Skeletal Attachment of a Femoral Prosthesis*. 2008

Studying the Bone

Biology

Biomechanics

Imaging

Bone

Bone Quality Assessment

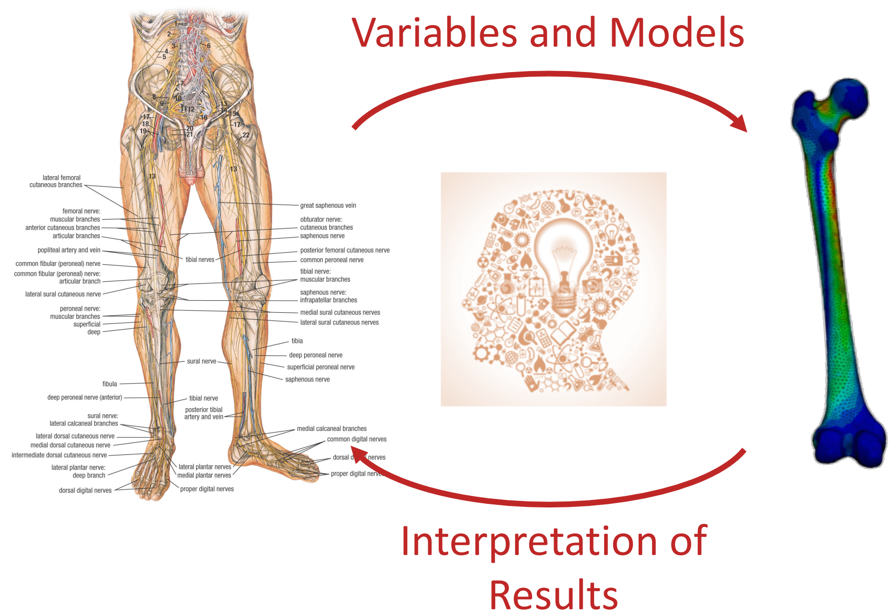
Bone Fracture and Fixation

Biomechanics Uses Models

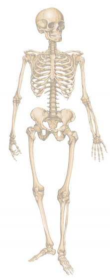
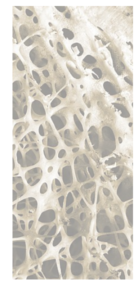
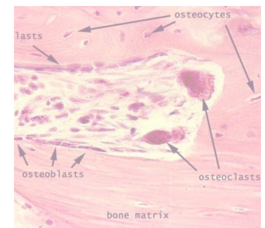
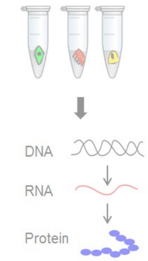
- Biomechanics is the application of classical mechanics to biological problems

- *Models* are:
    - the best tools we have to analyze physical problems
    - constructions of mind
    - simplifications of the real problem

- Warning 1: Do not confuse reality and model!
- Warning 2: GIGO!



Multi-Scale Approach to Bone Study

	BODY LEVEL	ORGAN LEVEL	TISSUE LEVEL	CELLULAR LEVEL	MOLECULAR LEVEL
Biology	 [m - cm]	 [cm - mm]			
Biomechanics	Rigid Object Models	Deformable Continuum Models		Statistical Models	
Imaging					

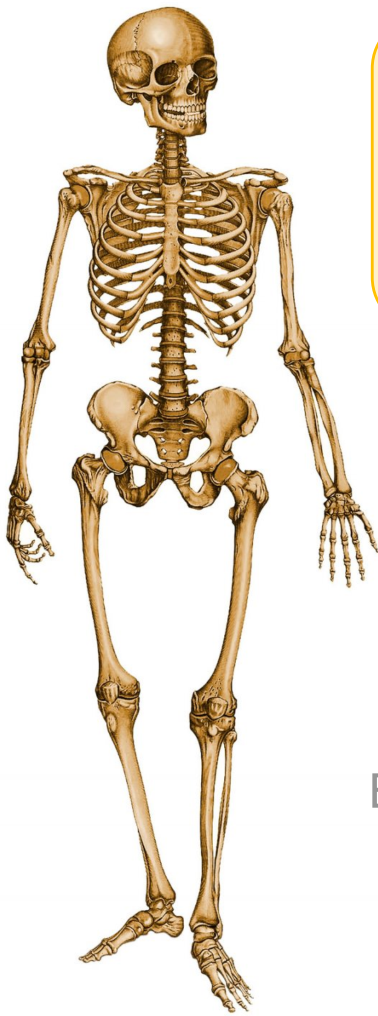
The Body Level

	BODY LEVEL	ORGAN LEVEL	TISSUE LEVEL	CELLULAR LEVEL
Biomechanics	Rigid Object Models	Deformable Continuum Models		Statistical Models

The Adult Skeleton

- The **adult** skeleton is made of 206 bones
- The functions of the skeleton are:

Protection
The cranium protects the brain, the rib cage protects heart and lungs



Movement
The bones and joints work with muscles to enable us to walk, run and sprint

Shape
Without the skeleton, the body would be flabby and shapeless

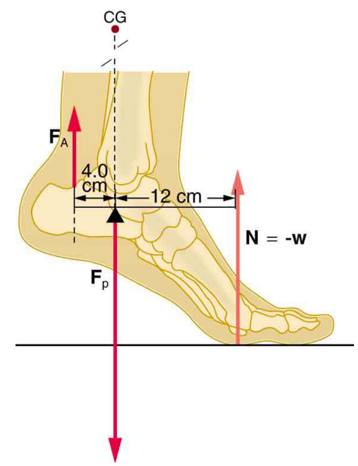
Blood Production
Red blood cells are made in the ribs and limb bones

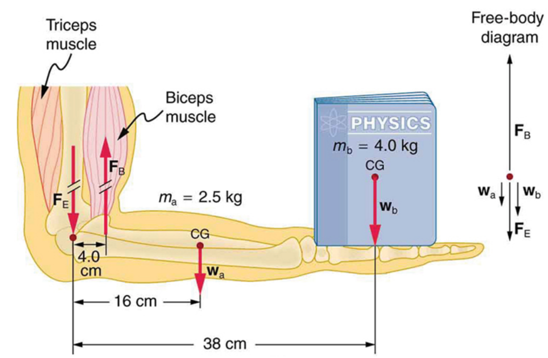
Support
The vertebrae support the head

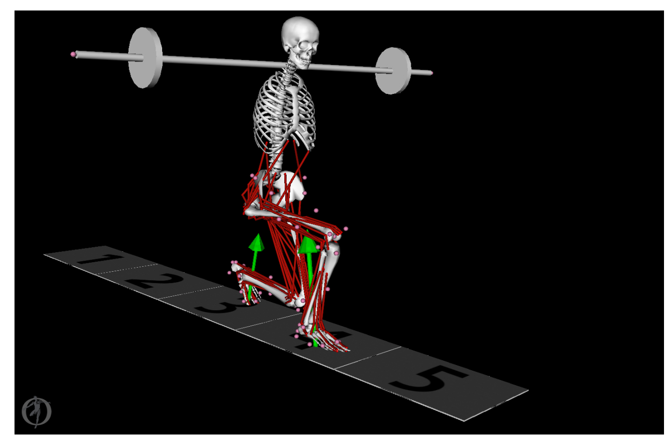
Calcium Storage
Bone is the largest supply of calcium

Kinematics - Force Analysis

- Skeleton is idealized as a kinematic chain of **rigid** objects
  - muscles are bundles of linear actuators connected to two bones
  - the rest of the body is considered only as mass lumped in selected points







The Organ Level

BODY LEVEL

ORGAN LEVEL

TISSUE LEVEL

CELLULAR LEVEL

Biomechanics

Rigid Object Models

Deformable Continuum Models

Statistical Models

The Adult Skeleton

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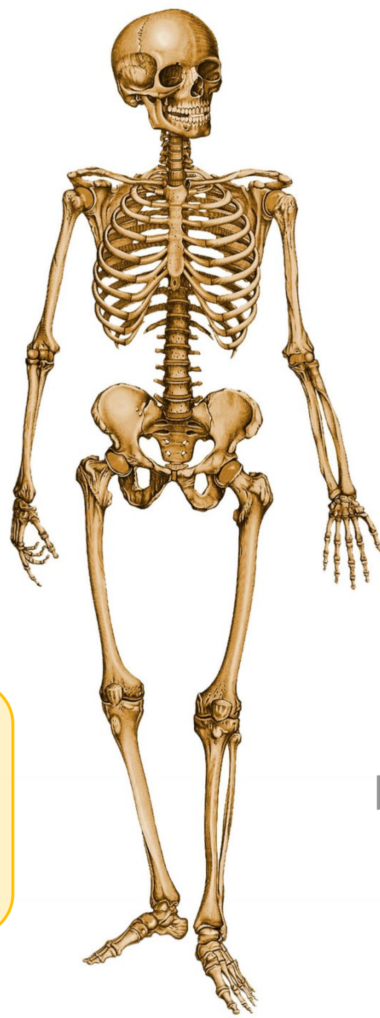
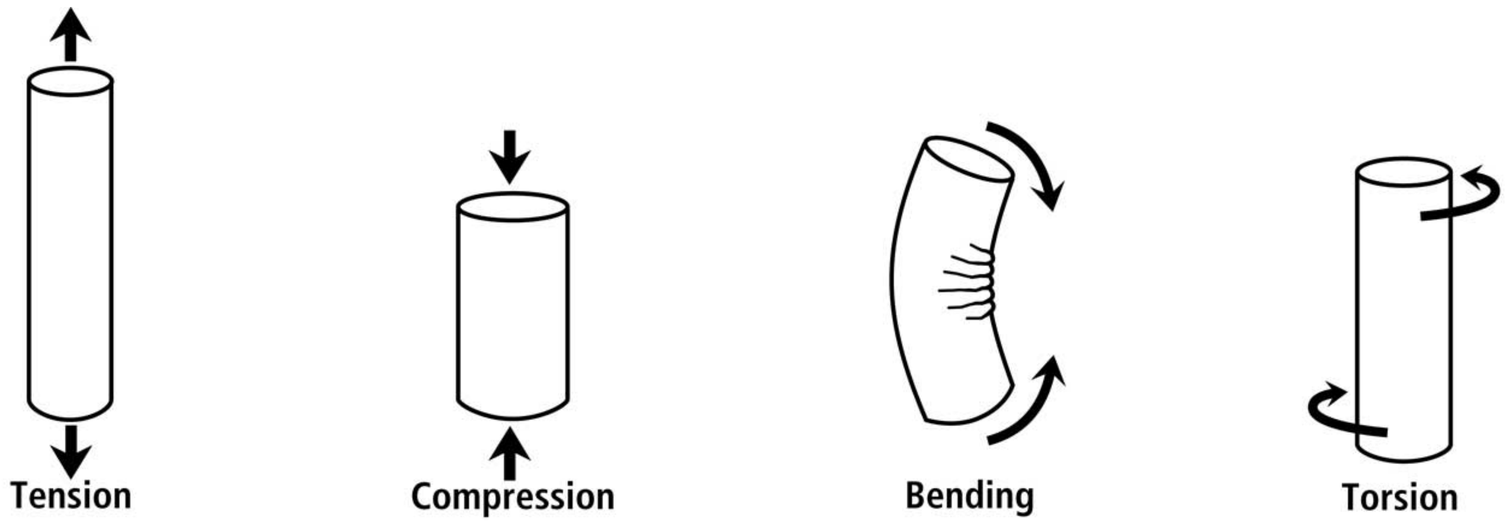


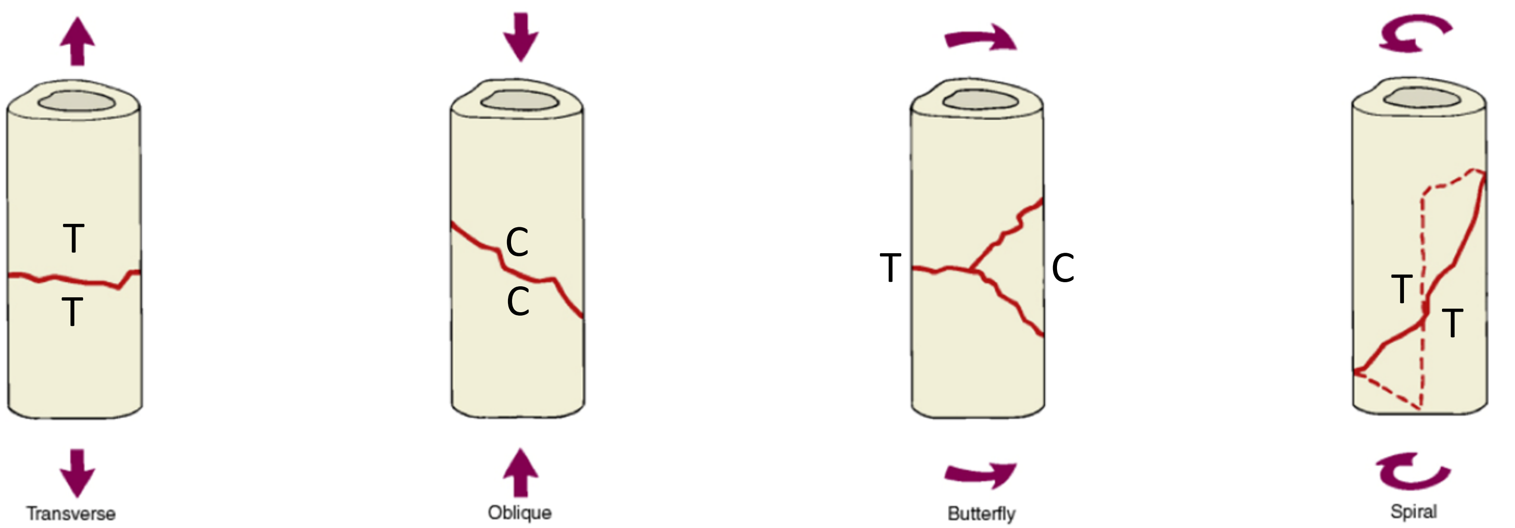
Movement
The bones and joints work with muscles to enable us to walk, run and sprint

Blood Production
Red blood cells are made in the ribs and limb bones

Calcium Storage
Bone is the largest supply of calcium

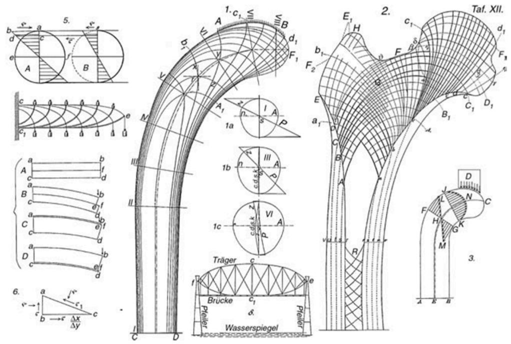
Forces Applied to Bones



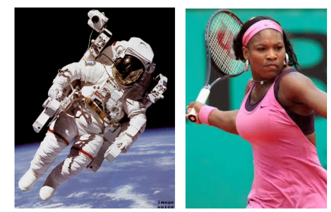


- How does bone respond to the applied forces?

Studying Bone Mechanics

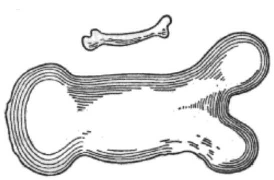


1867
Culmann and Von Meyer



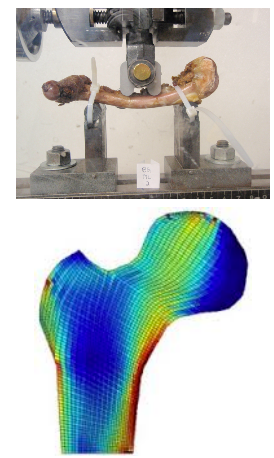
1960s
Frost's
Mechanostat
Regulation

1638
Galileo Galilei



1892
Wolff's Law

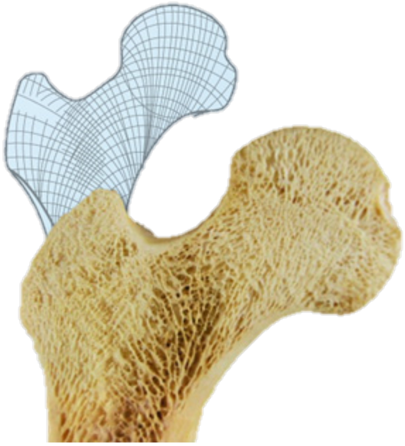
1970s-



Relationship Between Bone Function and Shape

- Wolff's law (Wolff, 1892)

  "Every change in the form and function of bone or of their function alone is followed by certain definite changes in their internal architecture, and equally definite alteration in their external conformation, in accordance with mathematical laws"



- Mechanostat regulation (Frost, 1960s)

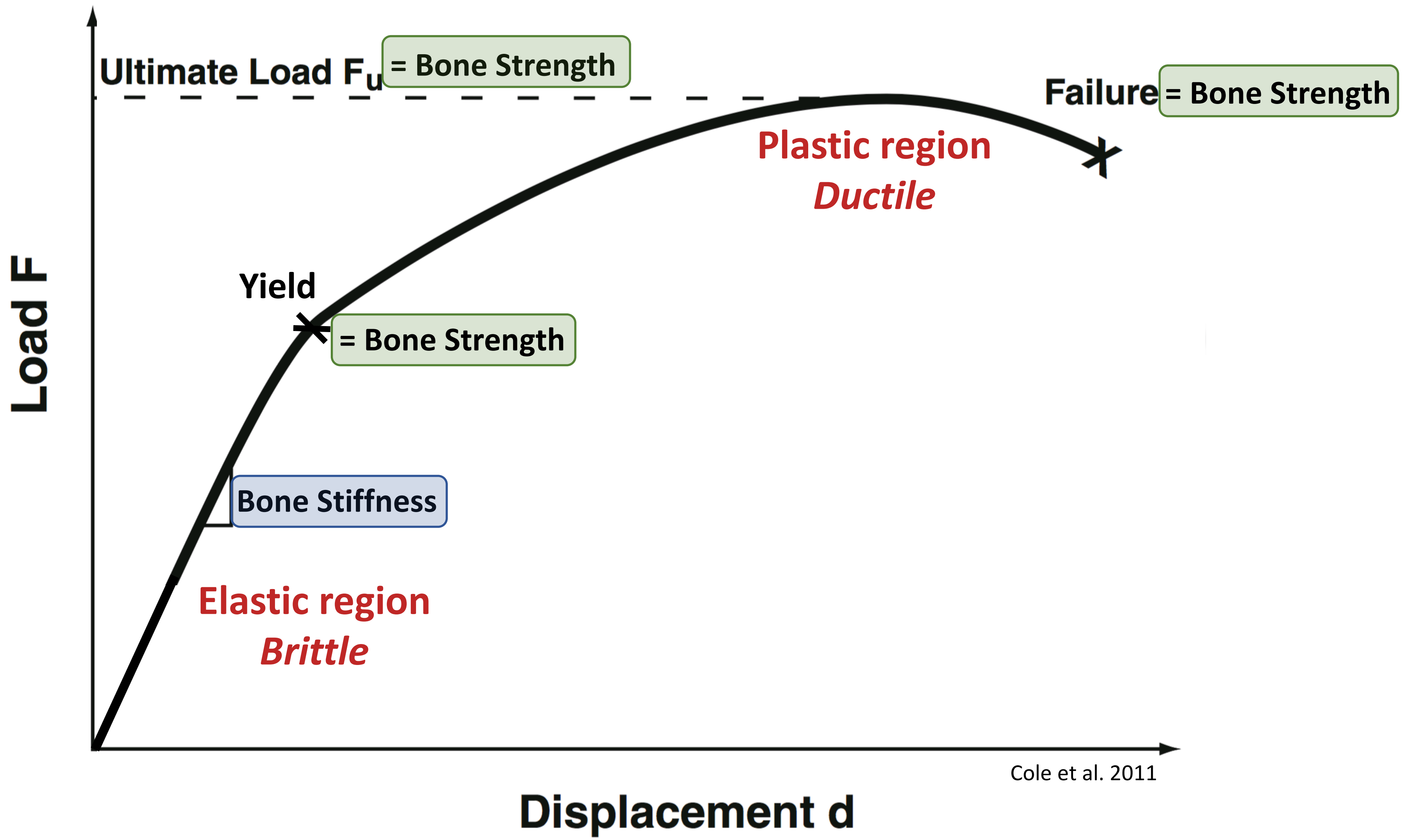
  The response of bone to its mechanical environment is controlled by a "mechanostat" that aims to keep bone tissue strain at an optimal level by homeostatically altering bone structure

Load-Deformation Test

- Example: Concrete Compression Test

https://www.youtube.com/watch?v=c7U91LbKFjI

Load-Deformation Curve

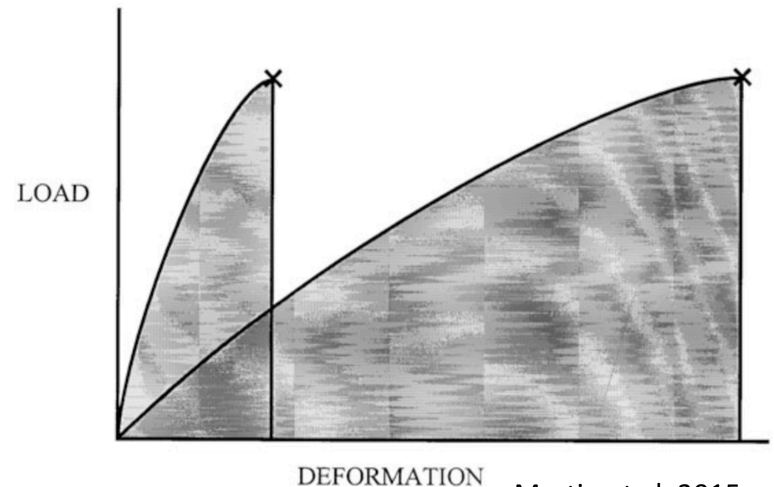


- *Stiffness* *[N/m]* = resistance to deformation
- *Strength* *[N]* = load at yield or failure or ultimate (conservative at yield)

Mechanical Failure of Whole Bones

The mechanical requirements of a bone are a compromise between:

- The need for stiffness to make muscle actions efficient
  - If bones are not stiff, part of muscle energy is used in deforming the bones, rather than creating useful motion
- The need for compliance to absorb energy and avoid fracture
  - More compliant (less stiff) bone is able to absorb a greater amount of energy before reaching the failure load



Martin et al. 2015

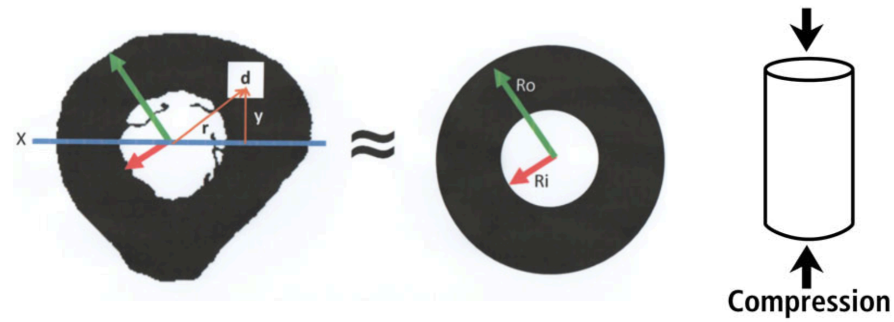
- The need for minimal skeletal weight
  - Mineral tissue is dense and requires high energy to be carried around

Bone Strength in Compression

www.youtube.com/watch?v=iF3RwVJglpc

Calculating Failure in Compression

- Let's consider a bone diaphysis as a cylinder with a periosteal diameter of 3 cm (R_o=1.5cm) and an endosteal diameter of 1.2 cm (R_i=0.6cm)
- Calculate failure load in *compression*



$$\text{Cross-sectional area} = A = \pi\left(R_p^2 - R_e^2\right) = \pi\left(0.015^2 - 0.006^2\right)$$

$$= 5.91 \times 10^{-4} \ \ \mathrm{m}^2.$$

ultimate stress in compression, $\boldsymbol{\sigma}_{\mathrm{f}} = 195$ MPa.

$$L_f = A\sigma_f = \left(5.94 \times 10^{-4}\right)\left(195 \times 10^6\right) = 1.16 \times 10^5 \ \ \mathrm{N}$$

- For a 70-kg person, this is about 170x body weight

1.16×10^5 N / (70Kg x 9.8N/Kg) = 169

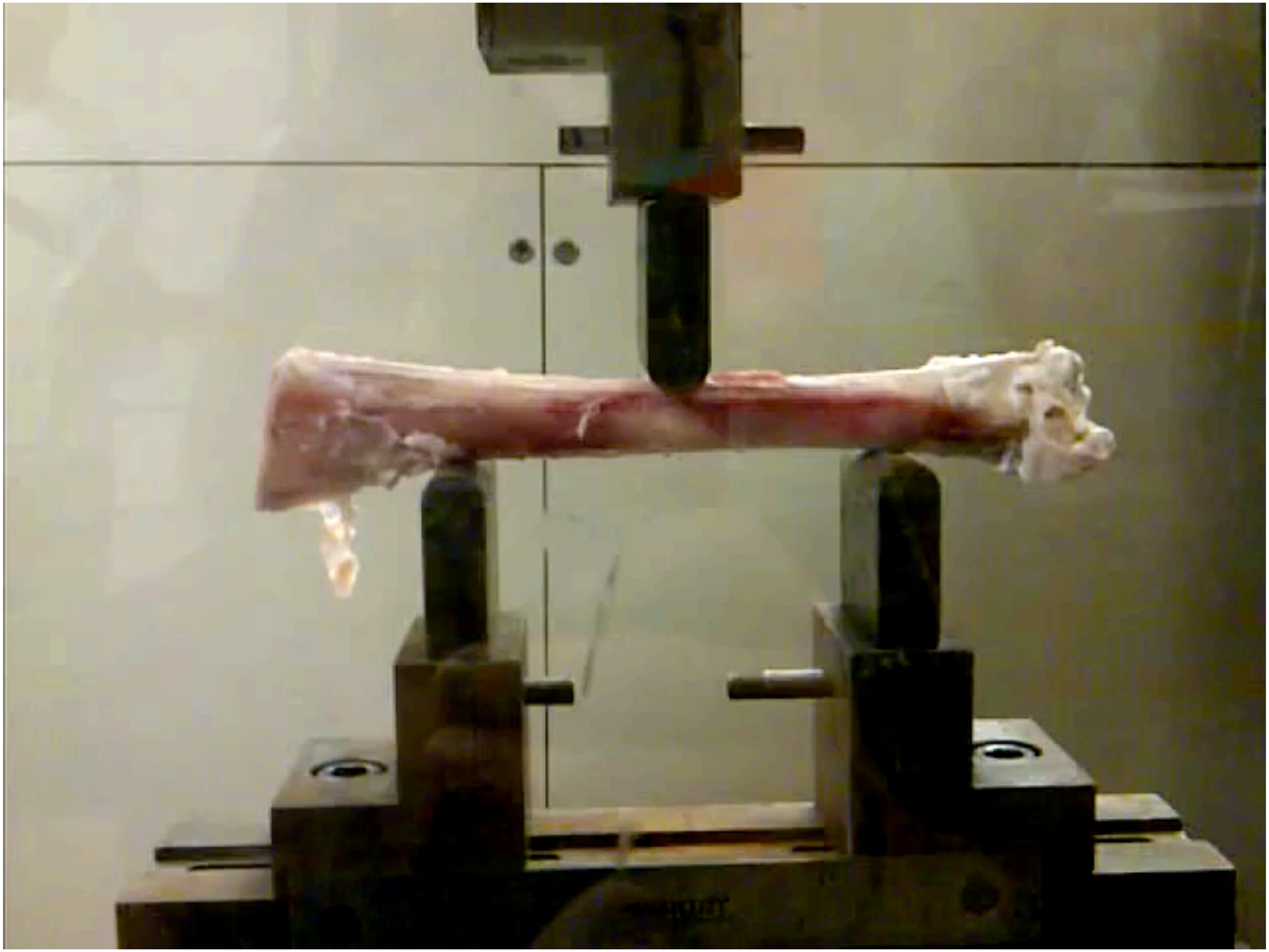
Table 7.1. Typical mechanical properties for cortical bone

Property	Human	Bovine
Elastic modulus, GPa		
Longitudinal	17.4	20.4
Transverse	9.6	11.7
Bending	14.8[a]	19.9[b]
Shear modulus, GPa	3.51	4.14
Poisson's ratio	0.39	0.36
Tensile yield stress, MPa		
Longitudinal	115	141
Transverse	—	—
Compressive yield stress, MPa		
Longitudinal	182	196
Transverse	121	150
Shear yield stress, MPa	54	57
Tensile ultimate stress, MPa		
Longitudinal	133	156
Transverse	51	50
Compressive ultimate stress, MPa		
Longitudinal	195	237
Transverse	133	178
Shear ultimate stress, MPa	69	73
Bending ultimate stress, MPa	208.6[a]	223.8[b]
Tensile ultimate strain		
Longitudinal	0.0293	0.0072
Transverse	0.0324	0.0067
Compressive ultimate strain		
Longitudinal	0.0220	0.0253
Transverse	0.0462	0.0517
Shear ultimate strain	0.33	0.39
Bending ultimate strain	–	0.0178[b]

Human data are for adult femur and tibia; bovine data are for femur
Data compiled from Cowin (1989, pp. 102, 103, 111–113), except as indicated
[a]From Currey and Butler (1975); adult femur, three-point bending
[b]From Martin and Boardman (1993); tibia, three-point bending

Martin et al. 2015

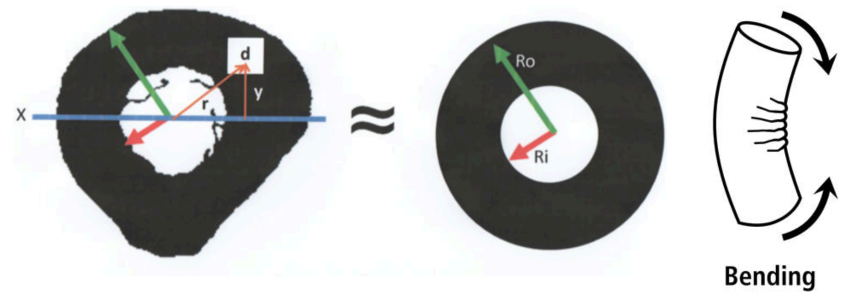
Bone Strength in Bending



https://www.youtube.com/watch?v=RI2h3iAyzH0

Calculating Failure in Bending

- Let's consider a bone diaphysis as a cylinder with a periosteal diameter of 3 cm (R_o=1.5cm) and an endosteal diameter of 1.2 cm (R_i=0.6cm)
- Calculate failure moment for *bending*



$$\text{Cross-sectional moment of inertia} = I = \pi\left(R_p^4 - R_e^4\right)/4 = \pi(0.015^4 - 0.006^4)/4$$

$$= 3.87 \times 10^{-8}\,\text{m}^4$$

ultimate stress in bending, $\sigma_{bf} = 208.6$ MPa

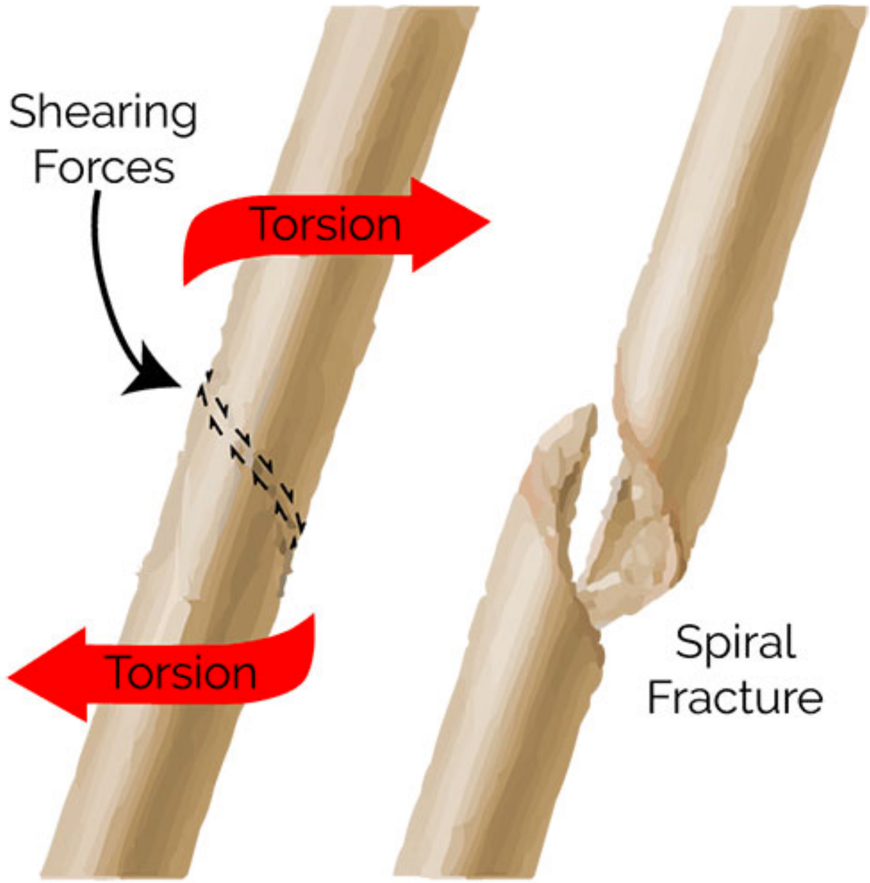
$$M_f = (I/c)\sigma_{tf} = \left(3.87 \times 10^{-8}/0.015\right)\left(208.6 \times 10^6\right) = 538\,\text{N} \cdot m$$

- For a 42cm femur supported at its ends, this M_f is produced by a force in the middle of 5124 N, which is 7x the body weight of a 70 kg person

F = 4M / L = 4x538 / 0.42) = 5124N

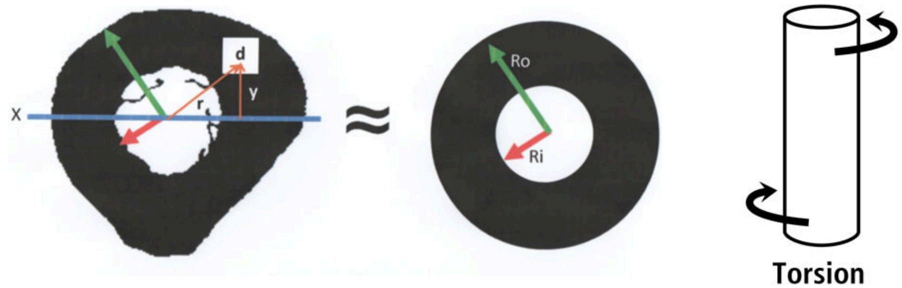
5124N / (70Kg x 9.8N/Kg) = 7.4

Bone Strength in Torsion



Calculating Failure in Torsion

- Let's consider a bone diaphysis as a cylinder with a periosteal diameter of 3 cm (R_o=1.5cm) and an endosteal diameter of 1.2 cm (R_i=0.6cm)
- Calculate failure moment for *torsion*



$$\text{Polar moment of inertia} = J = \pi\left(R_p^4 - R_e^4\right)/2 = \pi(0.015^4 - 0.006^4)/2$$
$$= 7.75 \times 10^{-8}\,\text{m}^4$$

ultimate shear stress, $\boldsymbol{\sigma}_{\text{sf}} = 69$ MPa

$$T_f = (J/r)\sigma_{sf} = \left(7.75 \times 10^{-8}/0.015\right)\left(69 \times 10^6\right) = 356N \cdot m$$

- For a femur supported at the distal end, this T_f is produced by a force at the head (b=9cm) of 3960N, which is 6x a body weight of a 70 kg person

$F = T_f$ / b = 356Nm / 0.09 = 3960N
3960 / (70Kg x 9.8N/Kg) = 5.7

Bone Geometry

- Let us assume *constant* bone *mass*

- How do we distribute bone mass to have stiff bones? Why do long bones have that shape?
  1. The epiphysis are *expanded* and filled with *trabecular* bone
  2. The shaft is a *thick-walled hollow* tube

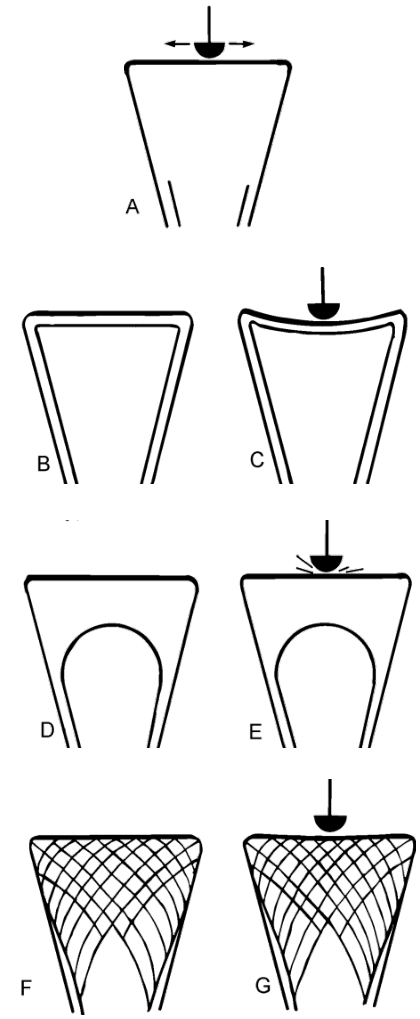
The Epiphysis Are *Expanded*…

- The forces across the joints are much larger than the forces in the shaft because of muscles
- The synovial cartilage is not a very strong tissue → large area for loads

Currey J. 2006

… and Filled With *Trabecular* Bone

- Load transmitted from cortex to cortex
- Thin cortex → Too large deformations
- Thick cortex → High local stresses
- Trabecular bone structure → Adequate overall small deformation

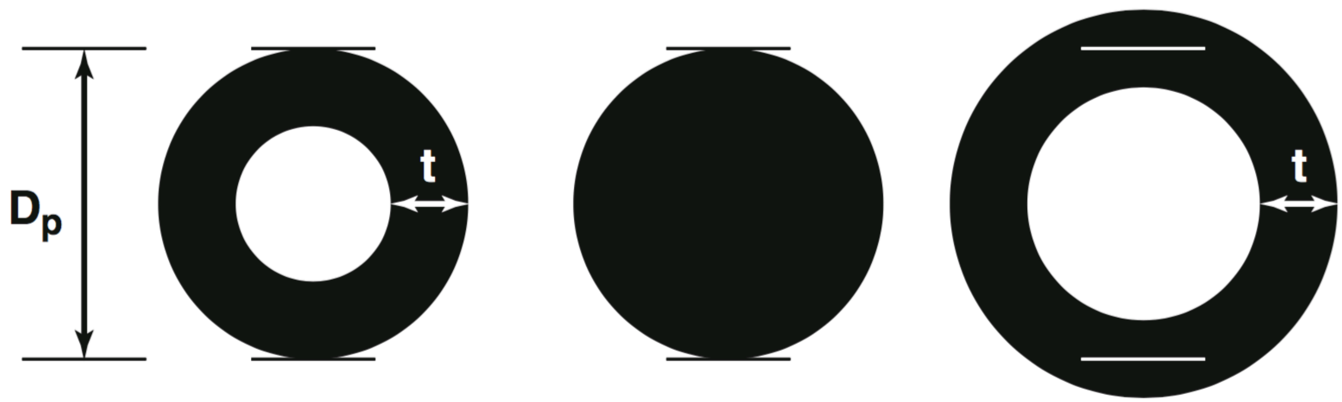


Currey J. 2006

The Shaft is a Thick-Walled Hollow Tube

- Bones are *hollow* to minimize the amount of material while maximizing resistance to load

Cross-sectional Resistance to Bending

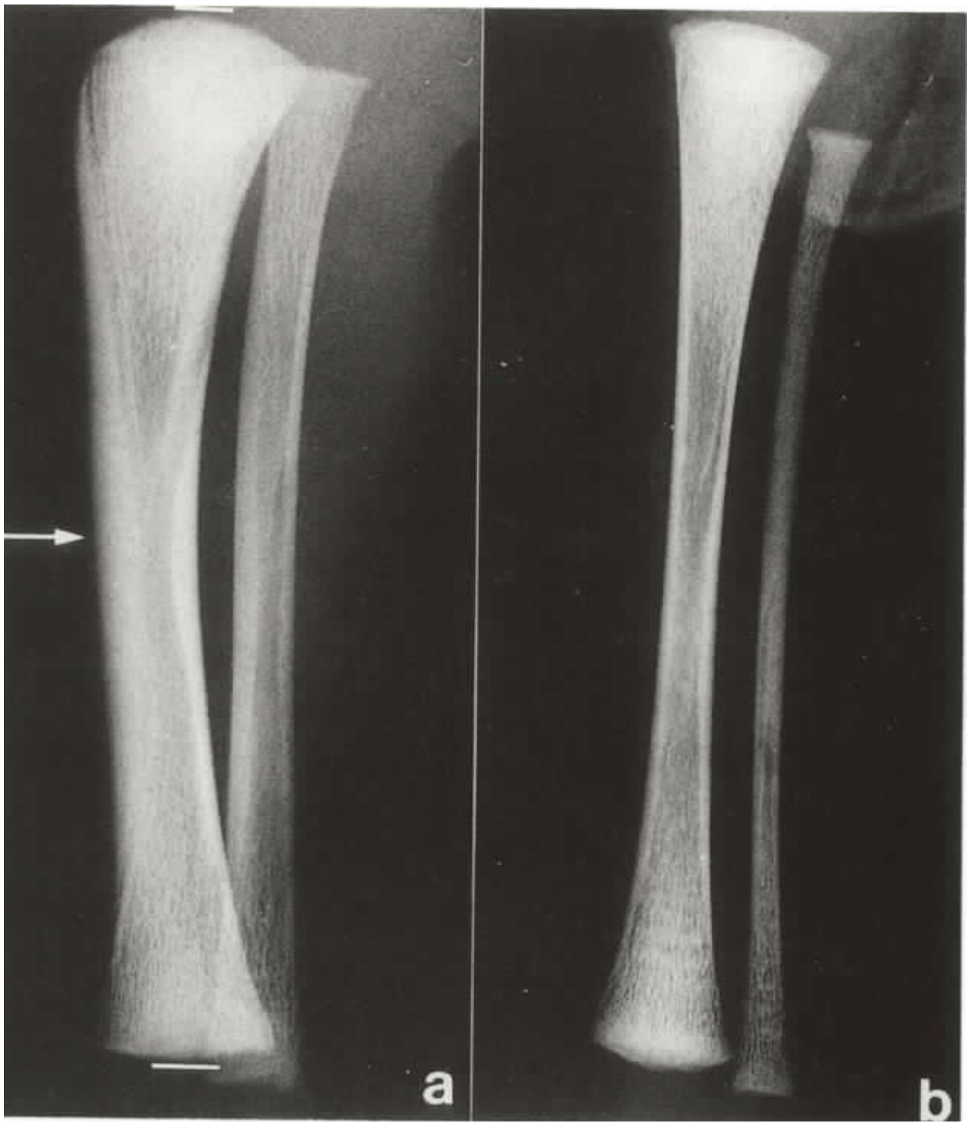


Periosteal Diameter (D_p):	100%	100%	125%
Cortical Area:	75%	100%	100%
Section Modulus:	94%	100%	170%

Cole et al. 2011

- Bone walls are *thick* to avoid *local buckling*

Bone Mechanics and Geometry - In Utero



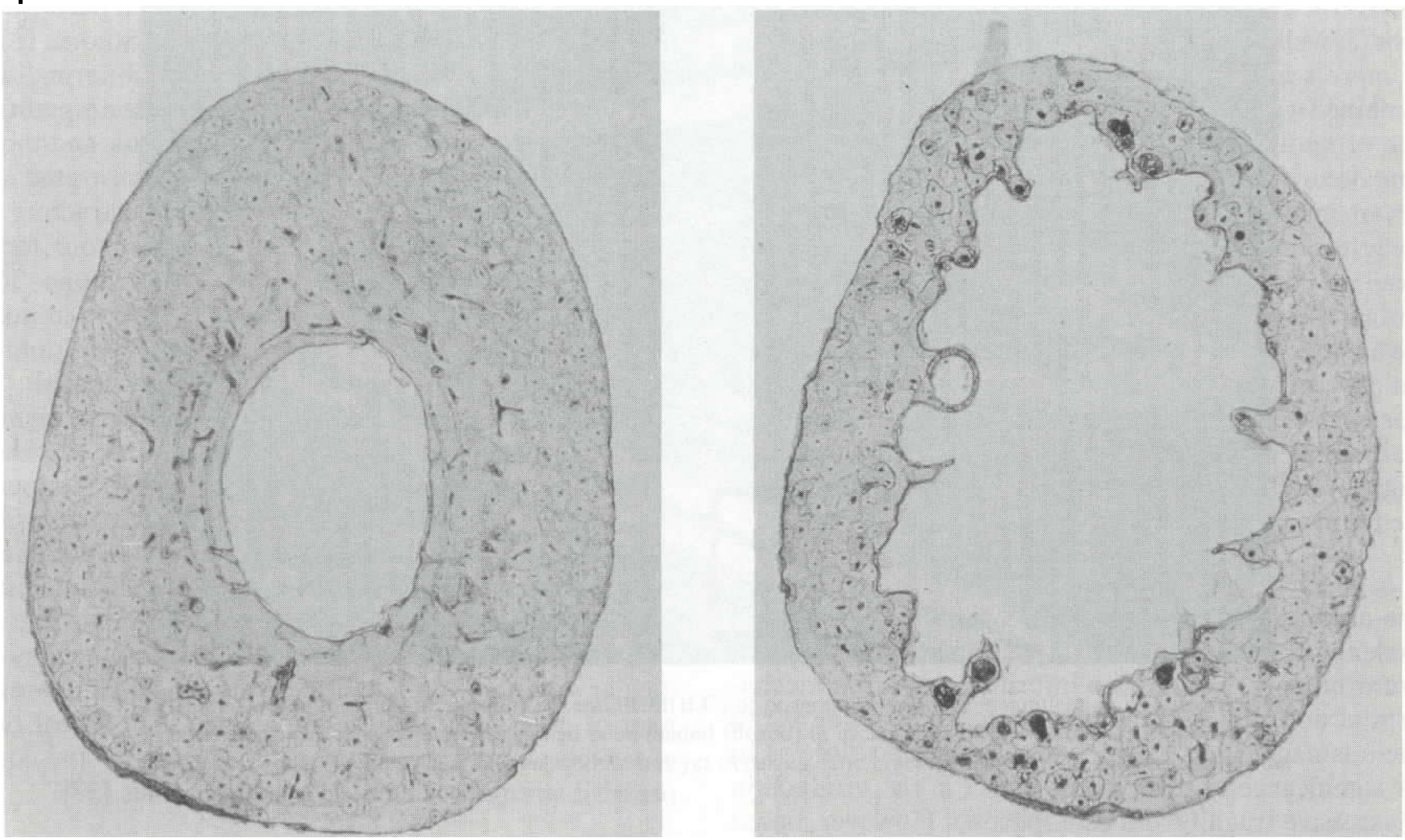
Rodriguez et al. 1988

- *In utero* hypokinesis leads to diminished bone diameter in newborns
- Appositional bone growth is regulated by diaphyseal surface strains *in utero [Van der Meulen et al. 1993]*

Courtesy of M.Levenston

Bone Mechanics and Geometry – Effects of Disuse

- Adult beagle humeri (bilateral) after 40 weeks of immobilization in a plaster cast



Courtesy of M.Levenston

Bone Mechanics and Bone Geometry: Long Duration Spaceflights

JOURNAL OF BONE AND MINERAL RESEARCH
Volume 21, Number 8, 2006
Published online on May 29, 2006; doi: 10.1359/JBMR.060509

Adaptation of the Proximal Femur to Skeletal Reloading After Long-Duration Spaceflight

Thomas F Lang,[1] Adrian D Leblanc,[2,3] Harlan J Evans,[4] and Ying Lu[1]

ABSTRACT: We studied the effect of re-exposure to Earth's gravity on the proximal femoral BMD and structure of astronauts 1 year after missions lasting 4–6 months. We observed that the readaptation of the proximal femur to Earth's gravity entailed an increase in bone size and an incomplete recovery of volumetric BMD.

Long Duration Spaceflights - Introduction

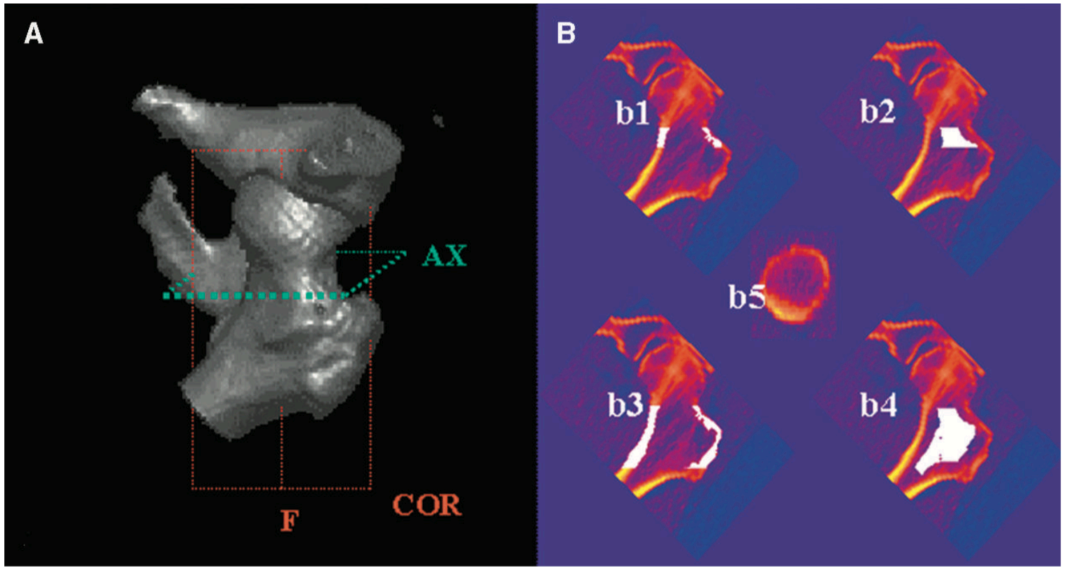
- Bone loss occurs during period of disuse
  - Documented for bedrest studies and subject with spinal cord injury
- Bone loss was hypothesized as a medical complication for astronauts
  - What is the effect of disuse and reloading on the bone mass of the proximal femur?
  - What is the effect on bone geometry?
  - What is the effect on bone strength?

Long Duration Spaceflights – Subjects and Methods

- 16 International Space Station Crewmembers
- Flight duration: 4.5-6 months

- QCT pre-flights, post-flight (within 3 weeks of landing), and after 1 year

- Measurements:
  - vBMD
  - Bone volume
  - Bone strength estimates



Adapted from T. Lang and G. Kazakia

Long Duration Spaceflights – Results and Conclusion

TABLE 1. FEMORAL NECK vBMD, MASS, AND VOLUME MEASURED PREFLIGHT (PRFL), POSTFLIGHT (POFL), AND 1 YEAR AFTER MISSION, FOLLOWED BY THE PERCENTAGE CHANGES MEASURED DURING THE FLIGHT AND RECOVERY PERIODS AND THE RATIO OF 1 YEAR TO PRFL VALUES

		PrFL	*PoFL*	*R12*	*Δ(%) Flight*	*Δ(%) Recovery*	*Ratio (R + 12/PreFL)*
Total vBMD (g/cm^3)	Mean	0.358	0.324	0.326	–9.40	0.90	0.91
	SD	0.054	0.051	0.046	6.40	5.90	0.05
	p value				<0.001	0.530	<0.001
Total volume (cm^3)	Mean	18.344	17.852	19.140	–1.40	7.20	1.05
	SD	3.545	2.642	3.401	10.90	9.90	0.12
	p value				0.500	0.020	0.180
MNCS (cm^2)	Mean	11.69	11.80	12.09	1.00	2.40	1.03
	SD	1.76	1.73	1.91	3.50	3.30	0.04
	p value				0.600	0.020	<0.001

TABLE 2. TOTAL FEMUR vBMD, MASS, AND VOLUME MEASURED PREFLIGHT (PRFL), POSTFLIGHT (POFL), AND 1 YEAR AFTER MISSION, FOLLOWED BY THE PERCENTAGE CHANGES MEASURED DURING THE FLIGHT AND RECOVERY PERIODS AND THE RATIO OF 1 YEAR TO PRFL VALUES

		PrFL	*PoFL*	*R12*	*Δ(%) Flight*	*Δ(%) Recovery*	*Ratio (R + 12/PreFL)*
Total vBMD (g/cm^3)	Mean	0.332	0.297	0.309	–10.40	4.40	0.93
	Median	0.325	0.297	0.308	–9.72	4.72	0.93
	p value				<0.001	0.02	<0.001
Total volume (cm^3)	Mean	106.322	104.242	112.008	-0.70	7.20	1.06
	SD	19.803	14.678	19.234	10.10	7.30	0.11
	p value				0.710	<0.001	0.070

- Bone geometry compensates for lack of bone density recovery

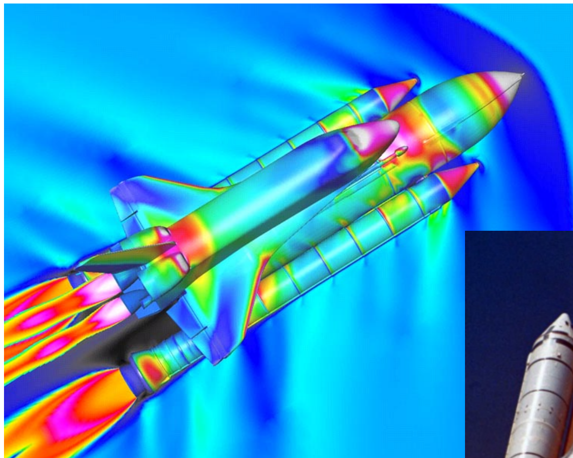
Adapted from T. Lang and G. Kazakia

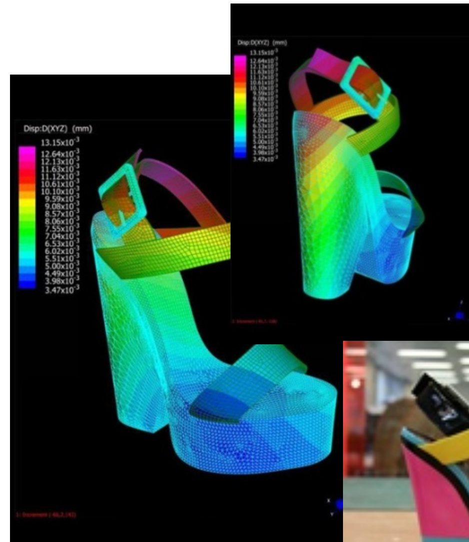
Finite Element Model (FEM)

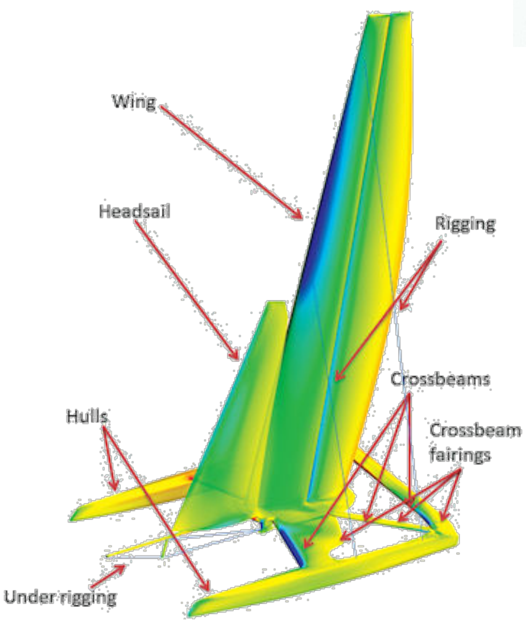
- The deformation of a solid body under the action of a system of external forces or displacements can we described using a system of partial differential equations (PDEs)
- Partial differential equations in a domain with complex geometry and external forces can be solved numerically using the Finite Element Model (FEM)

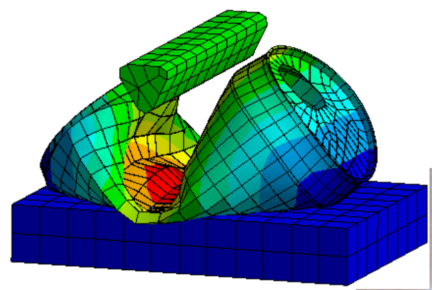
Creating the Model	Solving PDEs	Analysis and Interpretation of Results (Validation)
1. Geometry 2. Material Properties 3. Boundary Conditions		

Finite Element Models in *Engineering*

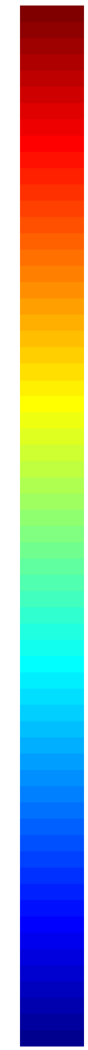






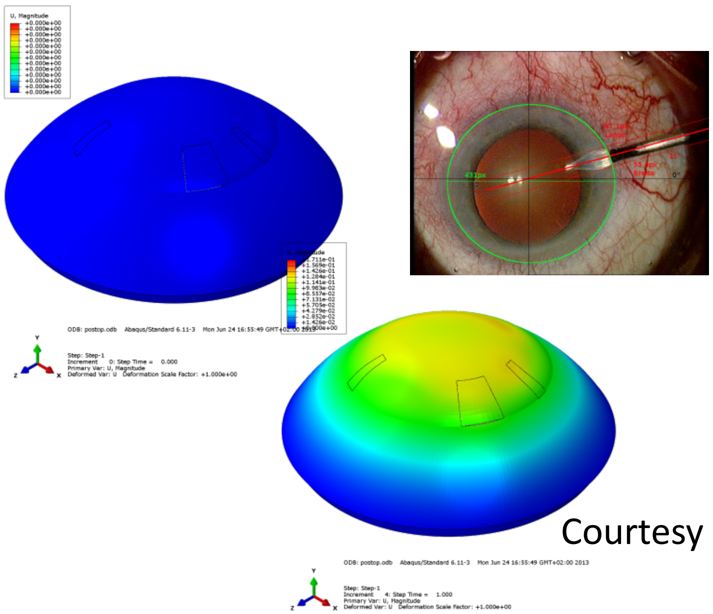


High

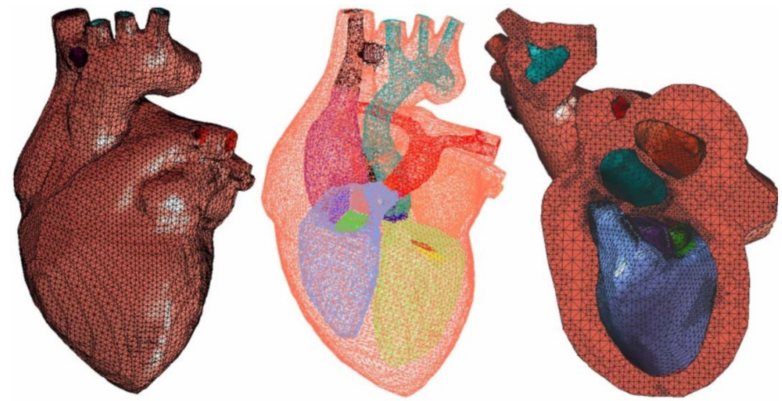


Low

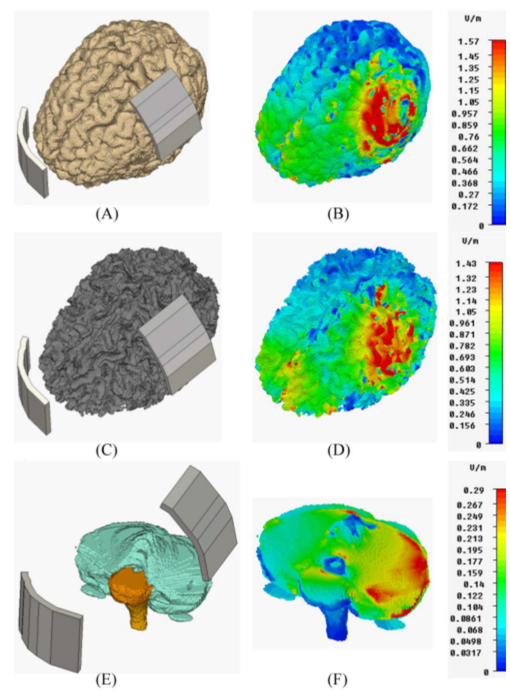
Finite Element Models in *Bioengineering*



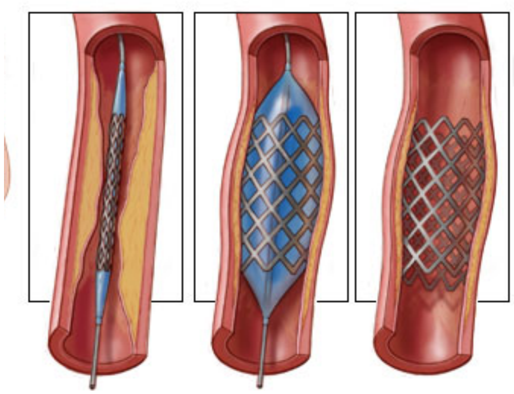
Courtesy H. Studer



Zhang et al. 2004



Parazzini et al. 2011



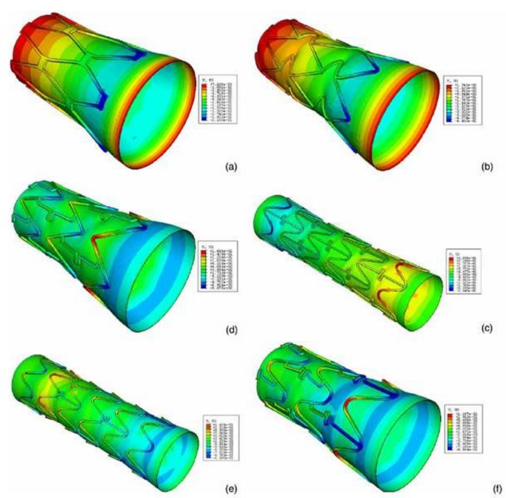
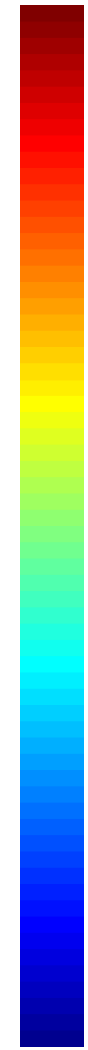


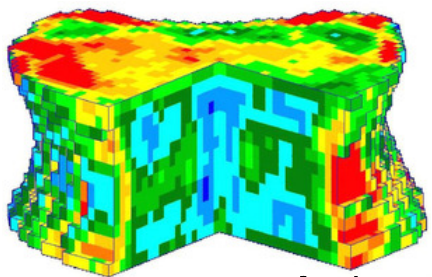
Figure 5. Displacement distributions for 6 different stent designs after balloon expansion [14]

High

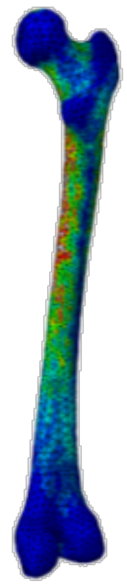


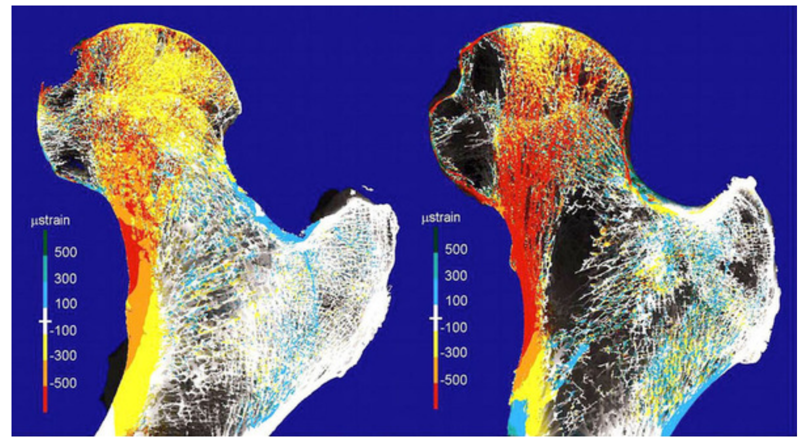
Low

Finite Element Models in *Bone*

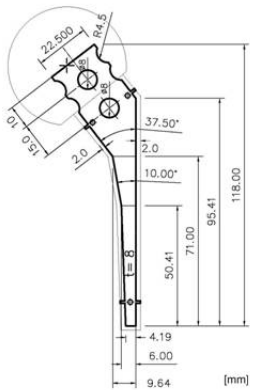


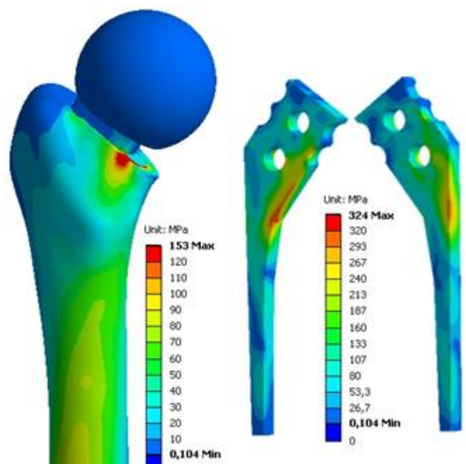
Crawford, 2003



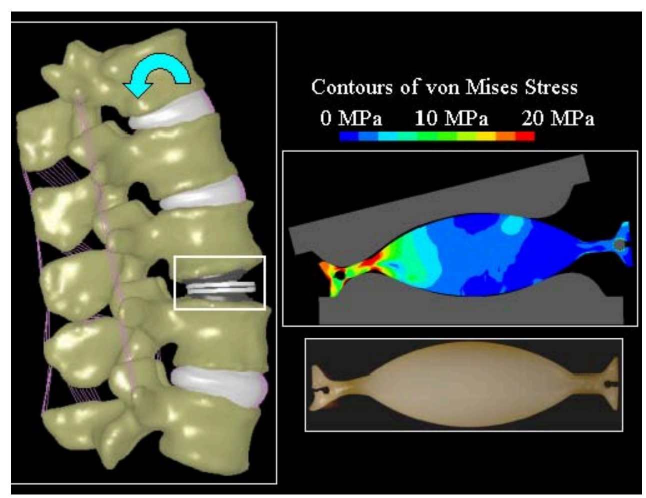


Van Rietbergen, 2003

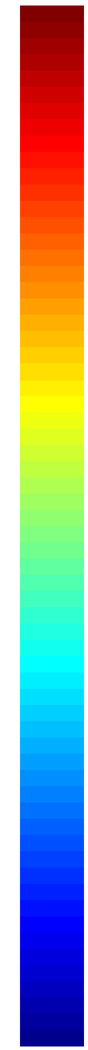




Thielen, 2009

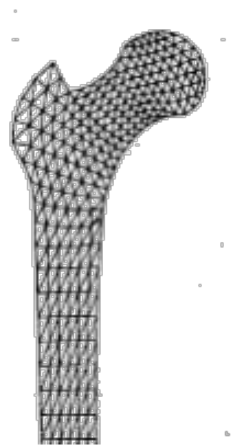


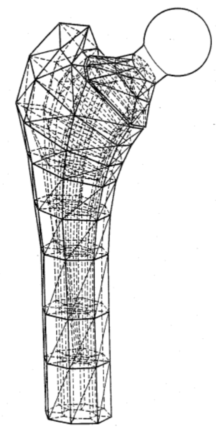
High

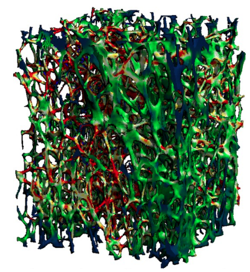


Low

Timeline of Finite Element Models in Bone







1947[1]
FEM
Mathematical
formalism

1972[2]
FEM in
Bone mechanics

1983[3]
FEM for
Stress analysis
Fracture fixation
Artificial joints

1993[4]
FEM for
Process simulations
Micro-structural
modeling

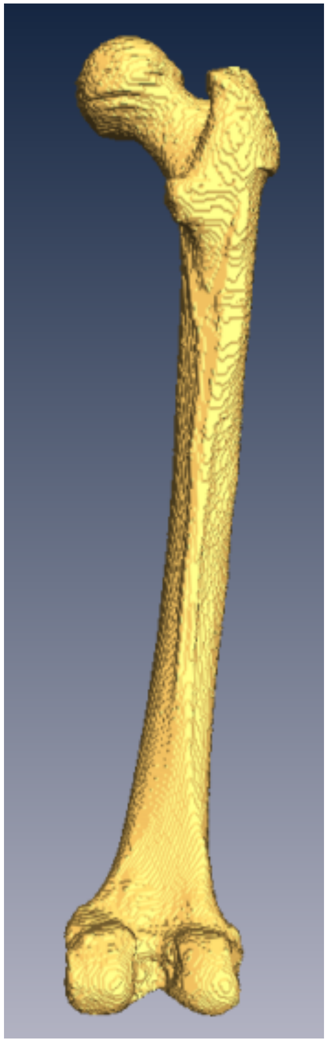
Today
Towards
Patient-specific
fracture risk

[1]http://en.wikipedia.org/wiki/Olgierd_Zienkiewicz
[2]Brekelmans W. et al. Acta orthop scand. 43 (5), 301–17. 1972.
[3]Huiskes R. J biomech. 16(6), 385-409. 1983.
[4]Huiskes R. J biomech eng. 115(4B), 520-527. 1993.

Geometry

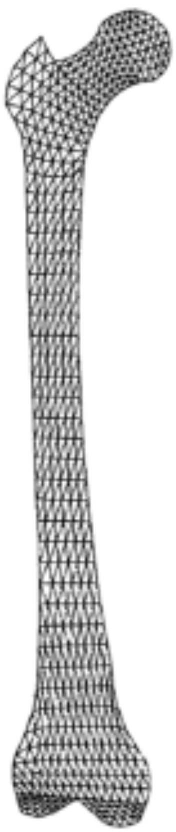
QCT

Segmentation

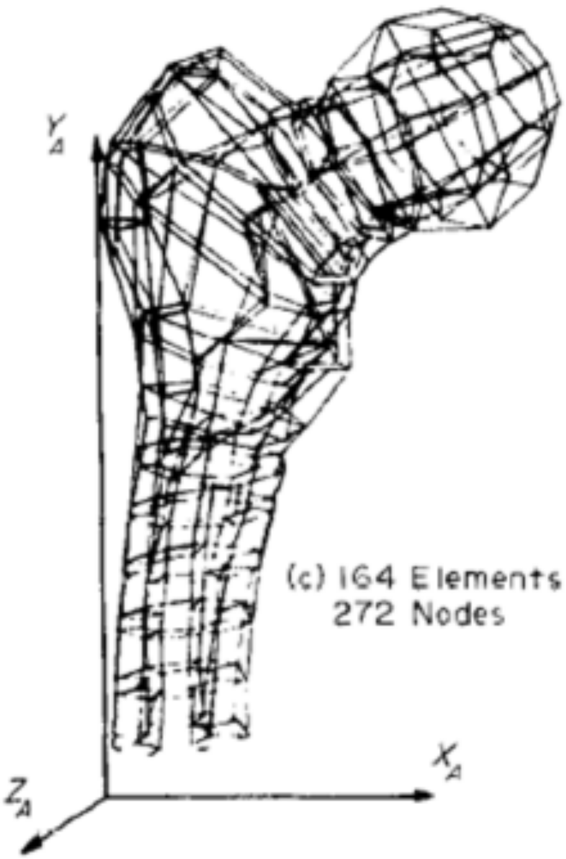


Meshing

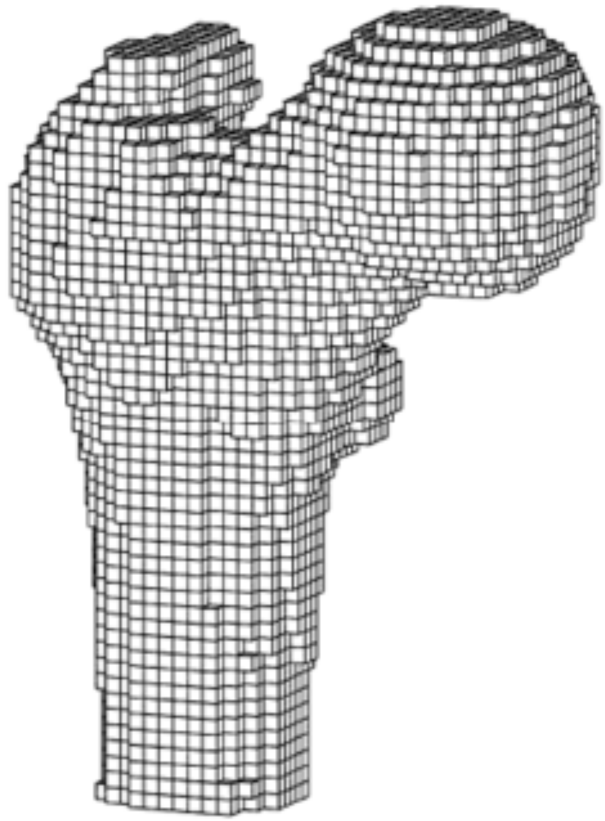
Meshing



Triangular surface mesh
[Brekelmans , 1972]



Eight-node volumetric mesh
[Villiappan, 1977]



Voxel-based mesh
[Keyak, 1990]

Tetrahedric mesh
[Viceconti, 1990]

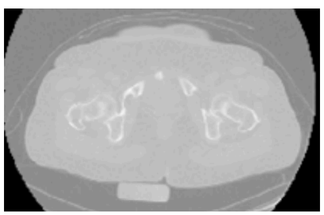
Material Properties

- Constant from experimental data
- Subject specific from images

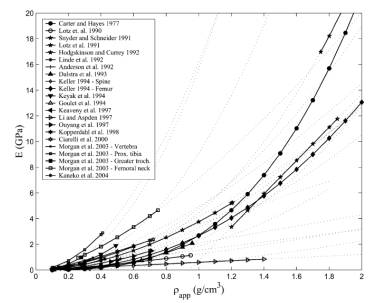
Bone Mineral Component | **Complete Bone**

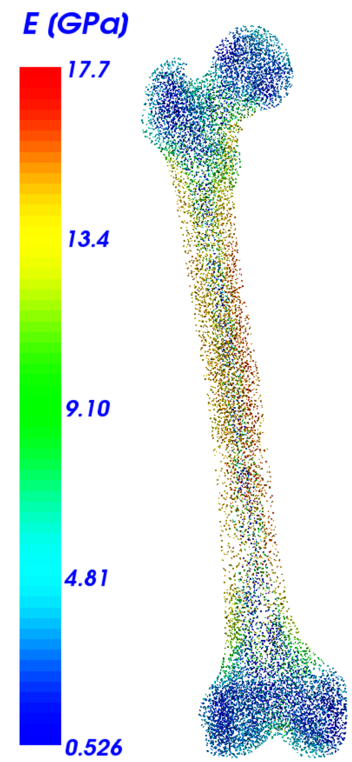
CT HU → ρ_{ash} → ρ_{app} → E

Calibration

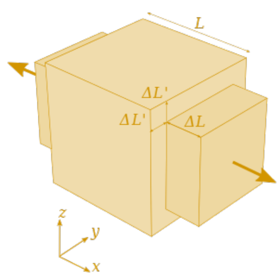


60%

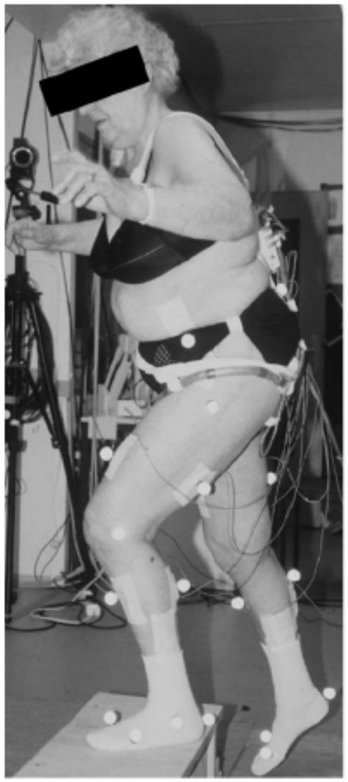




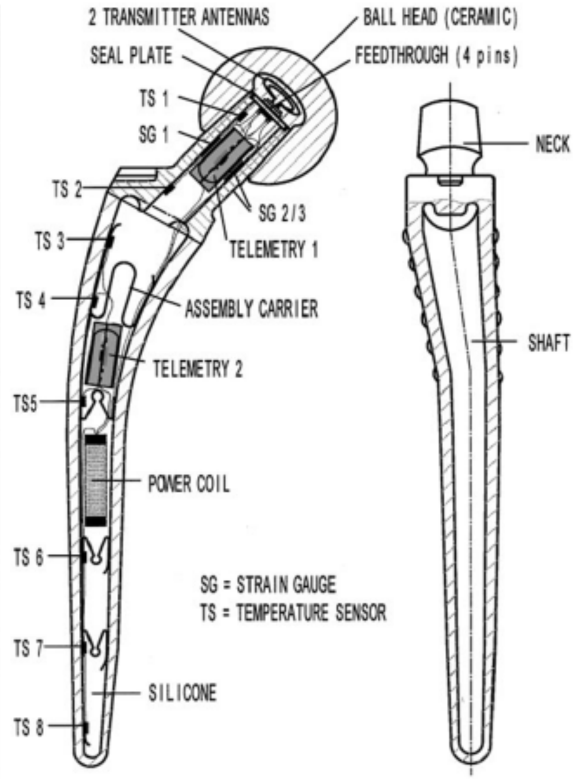
- Poisson's ratio = 0.3



Boundary Conditions



Gait analysis
[Heller,2001]



Instrumented hip implants
[Bergmann, 2008]

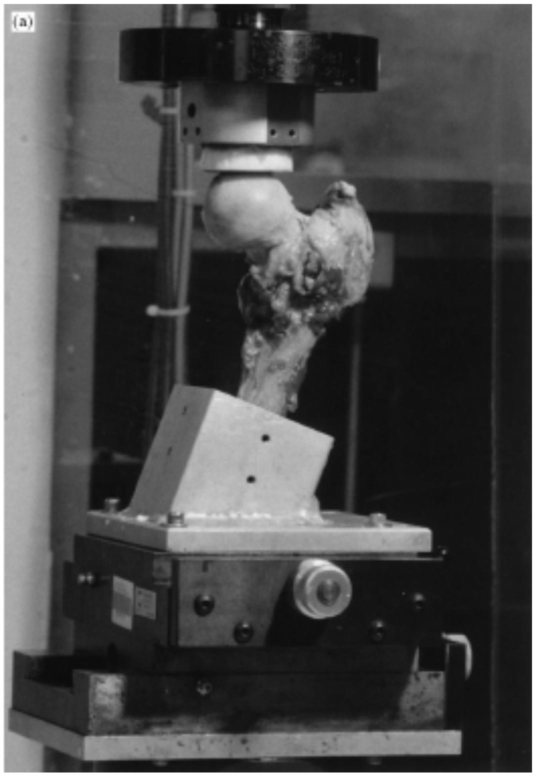
Simulation of interaction between bones and muscles
[Heller, 2001]

Validation

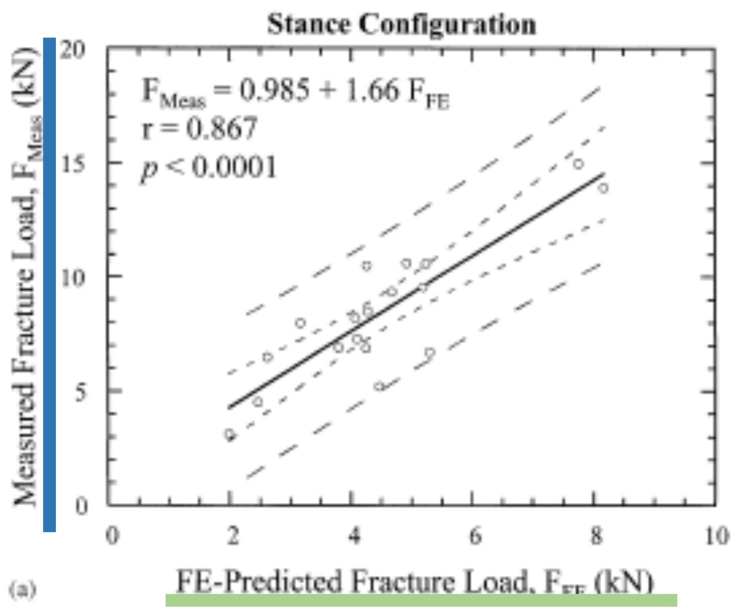
Creating the Model
1. Geometry
2. Material Prop.
3. Boundary Con.

Solving PDEs

Analysis and Interpretation of Results (Validation)



Experiment



FE simulation

[Keyak, 1998]

FEM to Estimate Femur Fracture Risk

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JOURNAL OF BIOMECHANICS

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www.JBiomech.com

Subject-specific finite element models implementing a maximum principal strain criterion are able to estimate failure risk and fracture location on human femurs tested *in vitro*

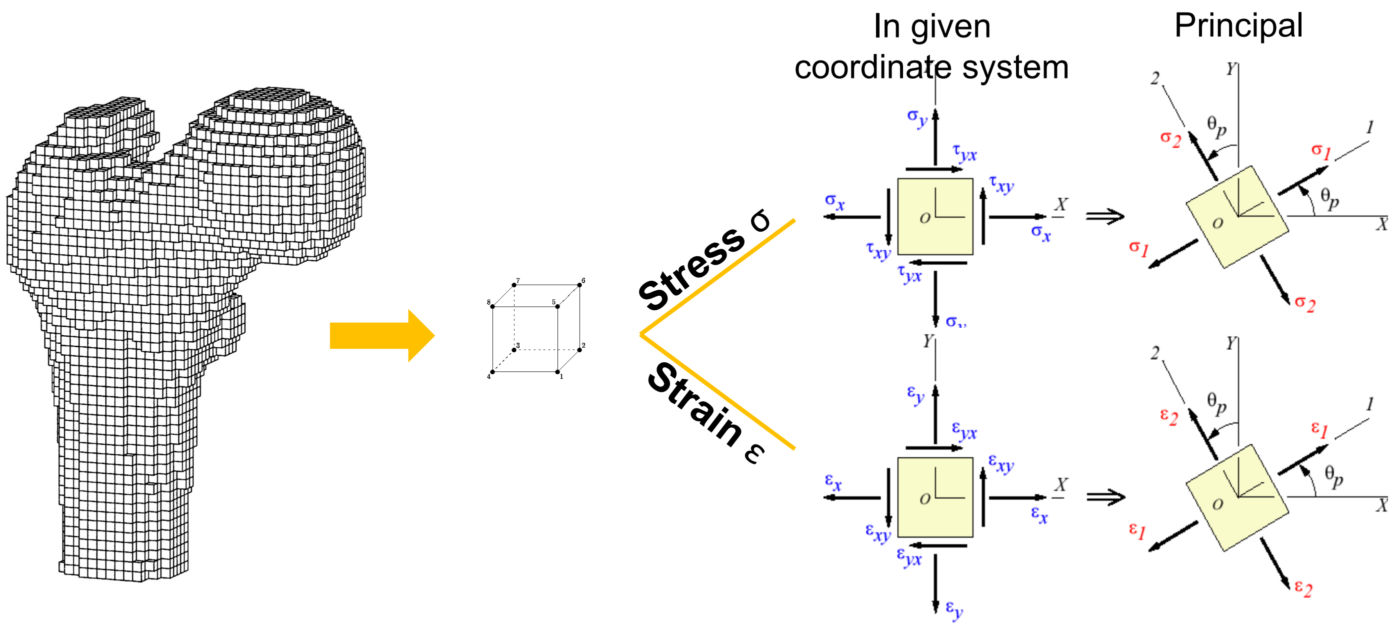
Enrico Schileo[a,*], Fulvia Taddei[a], Luca Cristofolini[a,b], Marco Viceconti[a]

[a]*Laboratorio di Tecnologia Medica, Istituti Ortopedici Rizzoli, Via di Barbiano 1/10, 40136 Bologna, Italy*
[b]*Engineering Faculty, University of Bologna, Italy*

Accepted 2 September 2007

Background – Stress, Strain and Failure Criteria

- Stress and strains are normalized forces and deformation of mesh elements



$\sigma_o = \sqrt{\sigma_I^2 - \sigma_I\sigma_{II} + \sigma_{II}^2}$

Max Principal Strain

- Brittle materials
- A material fails when the **Max Principal Strain** reaches the **"Limit" Strain**
- Tension: $\varepsilon_I > \varepsilon_L$ or $\varepsilon_{II} > \varepsilon_L$
- Compression: $|\varepsilon_I| > |\varepsilon_L|$ or $|\varepsilon_{II}| > |\varepsilon_L|$

Max Principal Stress

- Brittle materials
- A material fails when the **Max Principal Stress** reaches the **"Limit" Stress**
- Tension: $\sigma_I > \sigma_L$ or $\sigma_{II} > \sigma_L$
- Compression: $|\sigma_I| > |\sigma_L|$ or $|\sigma_{II}| > |\sigma_L|$

Von Mises Stress

- Ductile and Isotropic materials
- A material fails when when the **von Mises Equivalent Stress** (σ_o) exceeds the **Axial "Limit" Stress** (σ_{yL}): $\sigma_o = \sqrt{\sigma_I^2 - \sigma_I\sigma_{II} + \sigma_{II}^2} > \sigma_{yL}$

*Limit = yield, ultimate, fracture

FEM to Estimate Femur Fracture Risk - Introduction

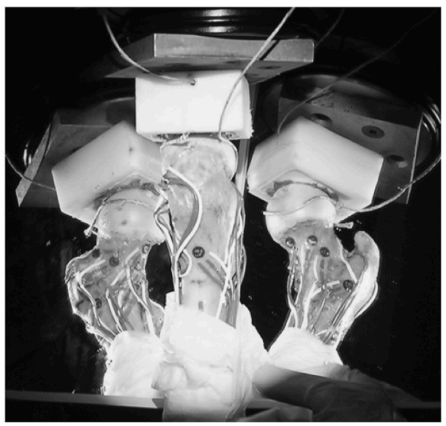
- No agreement on the choice of the failure criterion to adopt for the bone tissue can be found in the FEM literature
- The use of stress-based criteria prevails on strain-based ones, while basic bone biomechanics suggest using strain parameters to describe failure

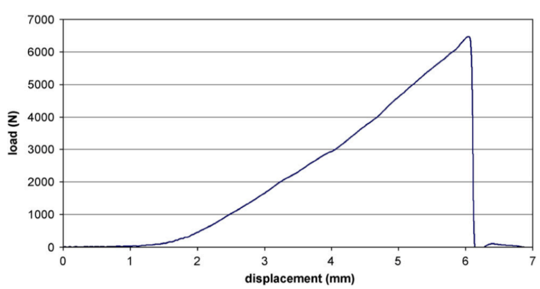
- Aim: To verify if a strain-based failure criterion can identify the failure patterns of bones in subject-specific simulations

FEM to Estimate Femur Fracture Risk - Method

Experimental Test

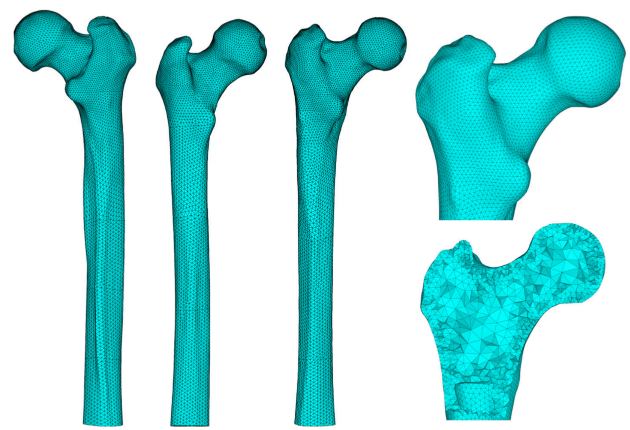
- Uniform load applied at 2mm/sec
- High-speed camera to observe the location of fracture onset





FE Simulation

- Geometry: from CT images
- Material properties: $E=6.950\rho_{app}^{1.49}$
- Force: first local maximum from load-deformation curve
- 3 different failure criteria:
  - Max principal strain (ε_{max})
  - Von Mises stress (σ_{VM})
  - Max principal stress (σ_{max})



FEM to Estimate Femur Fracture Risk - Failure Criteria

Table 3
Details of the failure criteria implemented

Criterion	Limit values	Implementation
ε_{max} (max principal strain)	Compressive: $\varepsilon^{C}_{lim} = 0.0104$;[a] Tensile: $\varepsilon^{T}_{lim} = 0.0073$[a]	1. Each element is assigned a tensile or compressive predominance: $\varepsilon_{max} = \sup(\lvert\varepsilon_1\rvert, \lvert\varepsilon_3\rvert)$ 2. The corresponding tensile or compressive ε_{lim} is chosen 3. RF is computed as $\boxed{RF = \varepsilon_{max}/\varepsilon_{lim}}$
σ_{max} (max principal stress)	Compressive: $\sigma^{C}_{lim} = 137\rho^{1.88}_{ash}$ ($\rho_{ash} < 0.317$ g/cm^3)[b] $\sigma^{C}_{lim} = 137\rho^{1.72}_{ash}$ ($\rho_{ash} < 0.317$ g/cm^3)[c] Tensile: $\sigma^{T}_{lim} = 0.8 * \sigma^{C}_{lim}$	1. σ^{C}_{lim} value is chosen upon element density, computed by BoneMat 2. Each element is assigned a tensile or compressive predominance $\sigma_{max} = \sup(\lvert\sigma_1\rvert, \lvert\sigma_3\rvert)$ 3. The corresponding tensile or compressive σ_{lim} is chosen 4. RF is computed as $\boxed{RF = \sigma_{max}/\sigma_{lim}}$
σ_{VM} (Von Mises stress)	$\sigma_{lim} = 137\rho^{1.88}_{ash}$ ($\rho_{ash} < 0.317$ g/cm^3)[b] $\sigma_{lim} = 137\rho^{1.72}_{ash}$ ($\rho_{ash} < 0.317$ g/cm^3)[c] No compressive/tensile distinction	1. σ_{lim} value is chosen upon element density, computed by BoneMat 2. RF is computed as $\boxed{RF = \sigma_{VM}/\sigma_{lim}}$

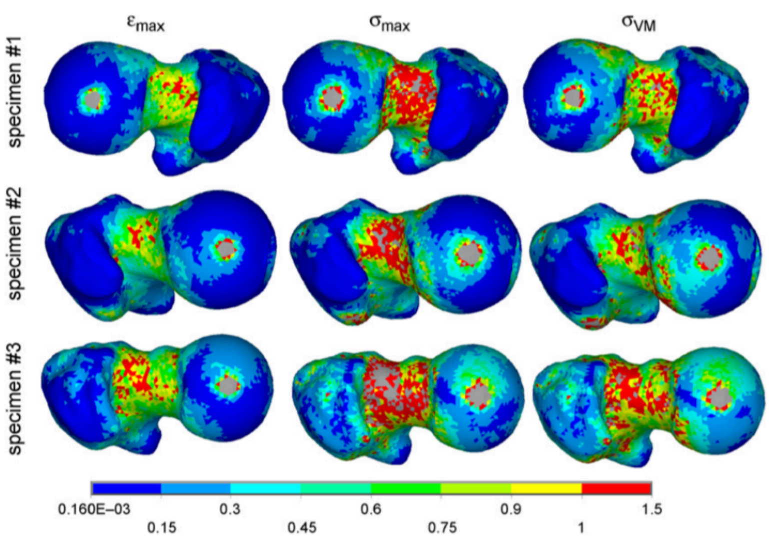
[a] Bayraktar et al. (2004b).
[b] Keyak et al. (1994).
[c] Keller (1994).

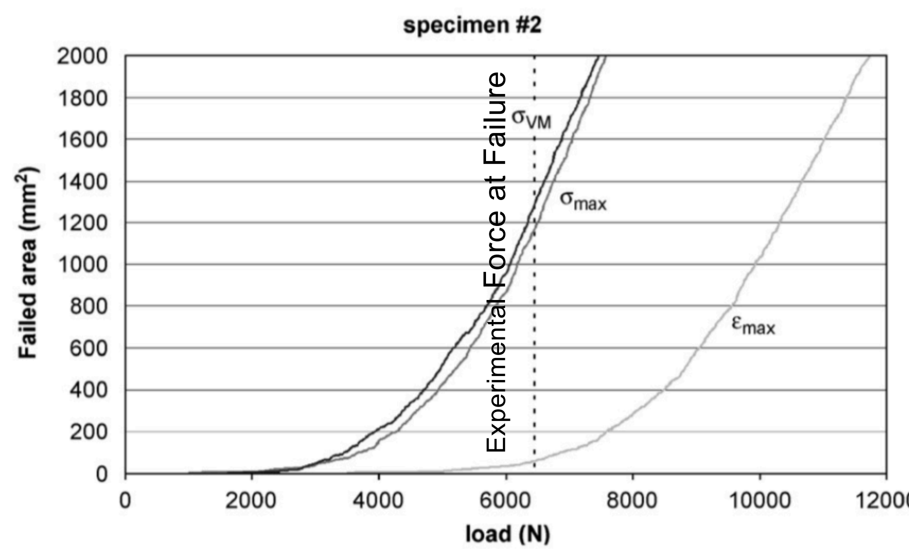
Note: Risk of Fracture = 1 / Factor of Safety

FEM to Estimate Femur Fracture Risk - Results

- All specimen failed for a tensile effect in the supero-lateral aspect of the neck
- All the FE models predicted a predominantly tensile stress at the actual failure locations

- The load at which the first superficial elements were expected to fail was close to the actual failure load for the ε_{max} RF





FEM to Estimate Femur Fracture Risk - Conclusion

- “The results seem to support, on a ‘‘whole bone’’ structural level and in clinically relevant loading conditions, the appropriateness of strain-based parameters to identify failure”
- Study suggests using Max Principle Strain Criteria to calculate subject-specific fracture risk using finite element models